The Equational Theories Project

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It aims at exploring equational theories of magmas.
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The project initially focused on problems of the form $A \Longrightarrow C$,
where A and C are $\forall$-quantified equations with at most four $\diamond$'s.

A Tricky Equational Problem

$$\begin{aligned} & \left(\forall x, y, z.\ x = x \diamond (y \diamond ((z \diamond x) \diamond y))\right) \\ \Longrightarrow\ & \left(\forall x, y, z.\ x = x \diamond (y \diamond (z \diamond (x \diamond z)))\right) \end{aligned}$$

The Vampire Proof (62 Steps)

1. ⊢ [X0,X1,X2] : op(X0,op(X1,op(op(X2,X0),X1))) = X0 [input]
2. ⊢ [X0,X1,X2] : op(X0,op(X1,op(X2,op(X0,X2)))) = X0 [input]
3. ⊢ [X0,X1,X2] : op(X0,op(X1,op(X2,op(X0,X2)))) = X0 [negated conjecture 2]
4. ? [X0,X1,X2] : op(X0,op(X1,op(X2,op(X0,X2)))) != X0 [ennf transformation 2]
5. ? [X0,X1,X2] : op(X0,op(X1,op(X2,op(X0,X2)))) != X0 => sK0 != op(sK0,op(sK1,op(sK2,op(sK0,sK2)))) [choice axiom]
6. sK0 != op(sK0,op(sK1,op(sK2,op(sK0,sK2)))) [skolemisation 4,5]
7. op(X0,op(X3,op(op(X2,X0),X3))) = X0 [cnf transformation 1]
8. sK0 != op(sK0,op(sK1,op(sK2,op(sK0,sK2)))) [cnf transformation 6]
9. op(X1,op(op(X2,X0),X1)) = op(op(X1,op(X2,X0)),X1),op(X3,op(X0,X3))) [superposition 7,7]
10. op(X1,op(op(X2,op(X3,op(X0,X1)),X2)),op(X0,X1)) = X1 [superposition 7,7]
11. op(op(X2,X0),X1) = op(op(op(X2,X0),X1),op(op(X1,op(X0,X3)),op(X1,op(op(X2,X0),X1)))) [superposition 10,7]
14. op(X7,op(op(op(op(X8,op(op(X9,op(X10,op(X5,op(X6,X7))))),X8)),X8)),X8)),op(X10,op(X5,op(X6,X7)))),op(X5,op(X6,X7))),op(X6,X7))) = X7 [superposition 10,10]
15. op(X1,op(op(X2,X0),X1)) = op(op(X1,op(op(X2,X0),X1)),op(X3,op(op(X4,X0),X3))) [superposition 10,7]
19. op(X3,op(X0,X3)) = op(op(X3,op(X0,X3)),op(X4,op(op(X1,op(op(X2,X0),X1)),X4))) [superposition 9,7]
21. op(X15,op(X11,op(op(X12,X11),X11))) = op(op(X15,op(op(X12,X13),X11)),X15),op(X16,op(op(X14,op(X13,X14)),X16))) [superposition 9,9]
22. op(X2,op(op(X3,op(op(X1,X0),X0)),X2)) = op(op(X2,op(op(X3,op(op(X4,X5),X1)),X2)),op(X0,op(op(X1,X0),X0))) [superposition 9,9]
34. op(X15,X16) = op(op(X15,X16),op(op(op(X13,op(op(X14,X15),X13)),X17),op(X16,op(X15,X16)))) [superposition 10,9]
35. op(X21,op(X20,X21)) = op(op(X21,op(X20,X21)),op(op(op(X23,op(X18,op(op(X19,X20),X18)),X22)),op(X18,op(op(X19,X20),X18)))) [superposition 10,9]
37. op(op(X1,op(op(X2,X0),X1)),X0) = op(op(X1,op(op(op(X2,X0),X1),X0)),op(X3,op(op(X4,op(op(X5,X0),X4)),X3))) [superposition 19,7]
53. op(X4,op(X2,X4)) = op(op(X4,op(X2,X4)),op(op(X3,op(X2,X3)),op(X0,op(op(X1,X2),X0)))) [superposition 19,15]
104. op(op(X3,X0),X4) = op(op(op(X3,X0),X4),op(op(op(X1,op(op(X2,X0),X1)),X0),op(X4,op(X3,X0),X4))) [superposition 11,7]
232. op(X35,op(X32,X35)) = op(op(X35,op(X32,X35)),op(op(op(X33,op(op(X34,X32),X33)),X32),op(X30,op(op(X31,X32),X30)))) [superposition 19,15]
237. op(X63,op(op(X66,op(op(X61,op(op(X62,X63),X61)),X66)),op(op(X64,op(op(X65,X62),X64)),X63))) = X63 [superposition 19,15]
269. op(X2,X4) = op(op(X2,X4),op(op(op(X3,op(X2,X3)),op(X0,op(op(X1,X2),X0))),op(X4,op(X2,X4)))) [superposition 34,9]
741. op(X2,op(op(op(X3,op(X2,X3)),op(X0,op(op(X1,X2),X0))),op(X4,op(op(X5,X2),X4)),X2))) = X2 [superposition 237,9]
896. op(X10,op(op(op(X8,X9),X9),X10)) = op(op(X10,op(op(op(X8,X9),X9),X10)),op(op(op(X8,X9),X9),op(op(op(X8,X9),X9),op(op(op(X8,X9),X9),op(X9,op(op(X8,X9),X9)))))) [superposition 29,11]
1791. op(op(X46,op(op(X44,X46),X46)),X43) = op(op(op(X46,op(op(X46,op(op(X44,X46),X46)),X43)),op(X39,op(op(X40,op(op(X41,X44),X40)),X42))),op(op(X46,op(op(X44,X46),X46)),X42))) [superposition 269,21]
1792. op(X48,op(op(X45,op(op(X46,X45),X45)),X48)) = op(op(X48,op(X45,op(op(X46,X45),X45),X48)),op(X44,op(X45,op(op(X46,X45),X45),X44))) [superposition 53,21]
1910. op(op(X40,op(op(X41,X40),X40)),X43) = op(op(op(X40,op(op(X41,X40),X40)),X39),op(op(X41,X40),X39))) [backward demodulation 1791,1792]
2162. op(X61,op(X58,X61)) = op(op(X61,op(X58,X61)),op(op(X62,op(op(X58,op(X59,op(op(X60,X58),X59)),X58),X62)),op(X58,op(op(X59,op(op(X60,X58),X59)),X58),X58),op(X58,op(op(X59,op(op(X60,X58),X59)),X58),X59))) [superposition 35,34]
2326. op(X61,op(X58,X61)) = op(op(X61,op(X58,X61)),op(op(X62,op(op(X59,op(op(X60,X58),X59)),X58),X62)),op(op(X59,op(op(X60,X58),X59)),X58),X59)) [backward demodulation 2162,2]
2615. op(X60,op(op(op(X66,op(op(X67,X68),X66)),X68),op(op(X61,op(op(X62,op(op(X63,X68),X62)),X61)),op(op(X64,op(op(X65,X68),X64)),X68))) = X68 [superposition 14,237]
2869. op(X29,op(X27,X29)) = op(op(X29,op(X27,X29)),op(op(X27,op(op(X28,X27),X27)),op(X27,op(op(X27,op(op(X28,X27),X27)),X27))))) [superposition 2326,29]
2886. op(op(X40,X39) = op(op(op(X40,X39),op(op(X44,op(op(op(X44,op(op(X43,X44),X44)),X40)),X41)),op(X39,op(X40,X39))))) [superposition 7,2326]
5621. op(X36,op(op(X37,op(op(X38,X31),X37)),X36)) = op(op(X36,op(op(X37,op(op(X38,X31),X37)),X36)),op(op(op(X32,op(X33,X31),X32)),X31),op(X34,op(op(X35,X31),X34))),op(X30,op(X31,X30)))) [superposition 21,232]
5844. op(X27,X29) = op(op(X27,X29),op(op(op(X27,op(op(X28,op(X27,X27)),op(X27,op(op(X28,X27),X27)))),op(X29,op(X27,X29))))) [superposition 2806,29]
5872. op(X0,op(op(X3,op(op(X0,op(op(X4,op(op(X5,X0),X4)),X5)),X3)),op(op(X1,op(op(X2,X0),X1)),X0))) = X0 [superposition 2806,7]
6230. op(op(X59,op(op(X64,X55),X59)),X55) = op(op(op(X59,op(op(X64,X55),X59)),X55),op(op(X56,op(op(X57,op(op(X58,op(X54,X55),X57)),X57)),X56)),op(X54,X55),X53))) [superposition 37,53]
10142. op(X0,op(op(X1,X0),X0)) = op(op(X0,op(op(X1,X0),X0)),op(X5,op(X0,op(op(X1,X0),X0),X5))) [superposition 1919,10]
10899. op(X985,op(op(op(X983,op(op(X984,X983),X983)),X983),op(op(X985,op(op(X986,op(op(X987,X985),X986)),X985)),X985))) = X985 [superposition 5872,1919]
10945. op(X1209,op(op(X1210,X1200),X1200)),X1211) = op(op(X1200,op(op(X1210,X1200),X1209)),X1211),op(op(X1212,op(op(X1213,X1211),X1212)),X1211),op(X1214,op(op(X1215,X1211),X1214))))) [superposition 232,1919]
10962. op(X27,X29)) = op(op(X27,X29),op(op(X27,op(op(X28,X27),X27)),op(X29,op(X27,X29)))) [backward demodulation 5844,10142]
10963. op(X29,op(X27,X29)) = op(op(X29,op(X27,X29)),op(X27,op(op(X28,X27),X27))) [backward demodulation 2869,10142]
10966. op(X27,X29) = op(op(X27,X29),op(X27,op(op(X28,X27),X27))) [backward demodulation 10962,9]
10435. op(X10,op(op(op(X8,X9),X9),X10)) = op(op(X10,op(op(op(X8,X9),X9),X10)),op(op(op(X8,X9),X9),op(op(op(X8,X9),X9),op(op(op(X8,X9),X9),op(X9,op(op(X8,X9),X9)))))) [backward demodulation 10966,10363]
10440. op(X10,op(op(op(X8,X9),X9),X10)) = op(op(X10,op(op(op(X8,X9),X9),X10)),op(X9,op(op(X8,X9),X9))) [backward demodulation 10435,9]
10453. op(X3,op(op(op(X0,op(X1,op(op(X2,X0),X1))),X3)) = op(op(X3,op(op(X0,op(X1,op(op(X2,X0),X1))),X3)),op(X1,op(op(X2,X0),X1))),X3)),op(X0,op(X1,op(op(X2,X0),X1))))) [superposition 10440,7]
10454. X3,op(X0,X3)) = op(op(X3,op(X0,X3)),op(X1,op(op(X2,X0),X1))) [forward demodulation 10453,7]
11071. op(X153,op(X151,X153)) = op(op(X153,op(X151,X153)),op(X151,op(X151,X151))) [superposition 53,10366]
11074. X160,op(X160,op(X160,op(X160,X160)),op(op(X162,op(op(X160,X160),X162)),X160))) = X160 [superposition 741,10366]
11290. op(X169,op(op(X168,op(X169,X168))) = X168 [forward demodulation 11074,10454]
11466. op(X,op(X7,X8)) = op(op(X8,op(X7,X8)),op(op(X9,op(op(X6,X7),X9)),op(op(X6,X7),op(op(X6,X7),op(X7,X9))))) [superposition 35,11290]
11690. X8,op(X7,X8)) = op(X8,op(X7,X8)),op(X5,op(op(X6,X7),X5))) [forward demodulation 11466,10871]
11691. op(X59,op(op(X60,X55),X59)),X55) = op(op(op(X59,op(op(X60,X58),X59)),X55),op(op(X56,op(op(X57,op(op(X58,X54),X55),X57)),X56)),op(X53,op(op(X54,X55),X53)))) [backward demodulation 6230,11690]
11716. op(op(X2,X8),X1) = op(op(op(X2,X8),X1),op(X3,op(X8,X3))) [backward demodulation 11,11690]
11722. op(X4,op(X2,X4)) = op(X4,op(op(X2,X4),op(X3,op(X2,X3)))) [backward demodulation 53,11690]
12256. op(X36,op(op(X37,op(op(X38,X31),X37)),X36)) = op(op(X36,op(op(X37,op(op(X38,X31),X37)),X36)),op(op(op(X32,op(op(X33,X31),X32)),X31),op(X34,op(op(X35,X31),X34))),op(X30,op(X31,X30)))) [backward demodulation 5621,11716]
12322. op(op(X59,op(op(X60,X55),X59)),X55) = op(op(X59,op(op(X60,X55),X59)),X55),op(X56,op(op(X54,X55),X56))) [backward demodulation 11699,11722]
12335. op(X3,X0),X4) = op(op(op(X3,X0),X4),op(op(X1,op(op(X2,X0),X1)),X0),op(X4,op(X3,X0),X4))) [backward demodulation 104,12322]
12421. op(op(X1209,op(op(X1210,X1200),X1200)),X1211) = op(op(X1200,op(op(X1210,X1200),X1209)),X1211),op(op(X1212,op(op(X1213,X1211),X1212)),X1211)) [backward demodulation 10345,12322]
12428. op(X36,op(op(X37,op(op(X38,X31),X37)),X36)) = op(op(X36,op(op(X37,op(op(X38,X31),X37)),X36)),op(op(X32,op(op(X33,X31),X32)),X31)) [backward demodulation 12256,12322]
12483. op(X60,op(op(op(X66,op(op(X67,X60),X66)),X68),op(op(X61,op(op(X62,op(op(X63,X68),X62)),X61)),op(op(X64,op(op(X65,X68),X64)),X68))) = X68 [backward demodulation 2615,12335]
12572. op(X60,op(op(op(X66,op(op(X67,X68),X66)),X68),op(X61,op(op(X62,op(op(X63,X68),X62)),X61))) = X68 [backward demodulation 12483,12428]
12659. op(X60,op(op(X66,op(op(X67,X68),X66)),X68)) = X68 [forward demodulation 12572,37]
12744. op(X985,op(op(op(X983,op(op(X984,X983),X983)),X985),op(op(X986,op(op(X987,X985),X986)),X985))) = X985 [backward demodulation 10899,12659]
12854. op(X985,op(op(X983,op(op(X984,X983),X983)),X985)) = X985 [forward demodulation 12744,12421]
12865. op(X48,op(op(X45,op(op(X46,X45),X45)),X48)) = op(op(X48,op(op(X45,op(op(X46,X45),X45)),X48)),X44) [backward demodulation 11792,12854]
13081. op(X48,X44) = X48 [forward demodulation 12865,12854]
14845. sK0 != op(sK0,sK1) [backward demodulation 8,13881]
14846. $false [subsumption resolution 14845,13881]

Tao's Equational Proof Challenge

Given that the Vampire proof is unintelligible, Terence Tao challenged the community to find "an alternate proof, by whatever means you wish—human, semi-automated, or automated."

Tao's Equational Proof Challenge Accepted

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Let’s Try Twee

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(137 Steps)

Introducing Krympa

Krympa

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Krympa is implemented in Rust, OCaml, and Python.

Krympa’s Proof for Tao’s Challenge (20 Steps)

```
class Magma (α : Type _) where
  op : α → α → α

infix:65 " ◇ " => Magma.op

abbrev Equation_a1 (G : Type _) [Magma G] :=
  ∀ x y z : G, x = (x ◇ (y ◇ ((z ◇ x) ◇ y)))

abbrev Equation_conjecture0 (G : Type _) [Magma G] :=
  ∀ x y z : G, x = (x ◇ (y ◇ (z ◇ (x ◇ z))))

theorem Equation_a1_implies_Equation_conjecture0 (G : Type _) [Magma G]
    (op_law : Equation_a1 G) : Equation_conjecture0 G :=
  have lemma_1 (x y z w : G) :
  (x ◇ ((y ◇ z) ◇ x)) = ((x ◇ ((y ◇ z) ◇ x)) ◇ (w ◇ (z ◇ w))) := by
    duper [op_law]

  have lemma_2 (x y z w v : G) :
  (x ◇ (y ◇ x)) = ((x ◇ (y ◇ x)) ◇ (z ◇ ((w ◇ ((v ◇ y) ◇ w)) ◇ z))) := by
    duper [lemma_1, op_law]

  have lemma_3 (x y z w v u : G) :
  (x ◇ ((y ◇ ((z ◇ w) ◇ y)) ◇ x)) =
      ((x ◇ ((y ◇ ((z ◇ w) ◇ y)) ◇ x)) ◇ (v ◇ ((u ◇ (w ◇ u)) ◇ v))) := by
    duper [lemma_1]

  have lemma_4 (x y z w v : G) :
  (x ◇ (y ◇ x)) = ((x ◇ (y ◇ x)) ◇ ((z ◇ (y ◇ z)) ◇ (w ◇ ((v ◇ y) ◇ w)))) := by
    duper [lemma_2, lemma_1]

  have lemma_5 (x y z w : G) :
  (x ◇ ((y ◇ ((z ◇ y) ◇ y)) ◇ x)) =
      ((x ◇ ((y ◇ ((z ◇ y) ◇ y)) ◇ x)) ◇ (w ◇ ((y ◇ ((z ◇ y) ◇ y)) ◇ w))) := by
    duper [lemma_4, lemma_3]

  have lemma_6 (x y z w : G) :
  ((x ◇ ((y ◇ x) ◇ x)) ◇ z) =
      (((x ◇ ((y ◇ x) ◇ x)) ◇ z) ◇ ((w ◇ ((x ◇ ((y ◇ x) ◇ x)) ◇ w)) ◇
        (z ◇ ((x ◇ ((y ◇ x) ◇ x)) ◇ z)))) := by
    duper [op_law, lemma_5]

  have lemma_7 (x y z w : G) :
  ((x ◇ ((y ◇ x) ◇ x)) ◇ z) =
      (((x ◇ ((y ◇ x) ◇ x)) ◇ z) ◇ (w ◇ ((x ◇ ((y ◇ x) ◇ x)) ◇ w))) := by
    duper [lemma_6, lemma_5]

  have lemma_8 (x y z w : G) :
  (((x ◇ ((y ◇ x) ◇ x)) ◇ z) ◇ ((x ◇ ((y ◇ x) ◇ x)) ◇ w)) =
      ((x ◇ ((y ◇ x) ◇ x)) ◇ z) := by
    calc
      (((x ◇ ((y ◇ x) ◇ x)) ◇ z) ◇ ((x ◇ ((y ◇ x) ◇ x)) ◇ w)) =
        (((x ◇ ((y ◇ x) ◇ x)) ◇ z) ◇ (((x ◇ ((y ◇ x) ◇ x)) ◇ w) ◇
          (((x ◇ ((y ◇ x) ◇ x)) ◇ ((x ◇ ((y ◇ x) ◇ x)) ◇ w)) ◇ ((w ◇
            ((x ◇ ((y ◇ x) ◇ x)) ◇ w)) ◇ ((x ◇ ((y ◇ x) ◇ x)) ◇ ((x ◇ ((y ◇ x) ◇ x)) ◇ w)))))) := by
        duper [op_law]
      (((x ◇ ((y ◇ x) ◇ x)) ◇ z) ◇ (((x ◇ ((y ◇ x) ◇ x)) ◇ w) ◇ (((x ◇ ((y ◇ x) ◇ x)) ◇ ((x ◇ ((y ◇ x) ◇ x)) ◇ w)) ◇
        ((w ◇ ((x ◇ ((y ◇ x) ◇ x)) ◇ w)) ◇ ((x ◇ ((y ◇ x) ◇ x)) ◇ ((x ◇ ((y ◇ x) ◇ x)) ◇ w)))))) =
        (((x ◇ ((y ◇ x) ◇ x)) ◇ z) ◇ (((x ◇ ((y ◇ x) ◇ x)) ◇ w) ◇ (((x ◇ ((y ◇ x) ◇ x)) ◇ ((x ◇ ((y ◇ x) ◇ x)) ◇ w)) ◇
          ((w ◇ ((x ◇ ((y ◇ x) ◇ x)) ◇ w)) ◇ ((x ◇ ((y ◇ x) ◇ x)) ◇ (((x ◇ ((y ◇ x) ◇ x)) ◇ w) ◇ (w ◇ ((x ◇ ((y ◇ x) ◇ x))
            ◇ w)))))))) := by
        duper [lemma_7]
      (((x ◇ ((y ◇ x) ◇ x)) ◇ z) ◇ (((x ◇ ((y ◇ x) ◇ x)) ◇ w) ◇ (((x ◇ ((y ◇ x) ◇ x)) ◇ ((x ◇ ((y ◇ x) ◇ x)) ◇ w)) ◇
        ((w ◇ ((x ◇ ((y ◇ x) ◇ x)) ◇ w)) ◇ ((x ◇ ((y ◇ x) ◇ x)) ◇ (((x ◇ ((y ◇ x) ◇ x)) ◇ w) ◇ (w ◇ ((x ◇ ((y ◇ x) ◇ x))
          ◇ w)))))))) =
        (((x ◇ ((y ◇ x) ◇ x)) ◇ z) ◇ (((x ◇ ((y ◇ x) ◇ x)) ◇ w) ◇ (((x ◇ ((y ◇ x) ◇ x)) ◇ ((x ◇ ((y ◇ x) ◇ x)) ◇ w)) ◇
          ((w ◇ ((x ◇ ((y ◇ x) ◇ x)) ◇ w)) ◇ (((x ◇ ((y ◇ x) ◇ x)) ◇ (((x ◇ ((y ◇ x) ◇ x)) ◇ w) ◇ (w ◇ ((x ◇ ((y ◇ x) ◇ x))
            ◇ w)))) ◇ ((((x ◇ ((y ◇ x) ◇ x)) ◇ w) ◇ (w ◇ ((x ◇ ((y ◇ x) ◇ x)) ◇ w))) ◇ ((x ◇ ((y ◇ x) ◇ x)) ◇ (((x ◇ ((y ◇ x)
              ◇ x)) ◇ w) ◇ (w ◇ ((x ◇ ((y ◇ x) ◇ x)) ◇ w)))))))))) := by
        duper [lemma_7]
      (((x ◇ ((y ◇ x) ◇ x)) ◇ z) ◇ (((x ◇ ((y ◇ x) ◇ x)) ◇ w) ◇ (((x ◇ ((y ◇ x) ◇ x)) ◇ ((x ◇ ((y ◇ x) ◇ x)) ◇ w)) ◇
        ((w ◇ ((x ◇ ((y ◇ x) ◇ x)) ◇ w)) ◇ (((x ◇ ((y ◇ x) ◇ x)) ◇ (((x ◇ ((y ◇ x) ◇ x)) ◇ w) ◇ (w ◇ ((x ◇ ((y ◇ x) ◇ x))
          ◇ w)))) ◇ ((((x ◇ ((y ◇ x) ◇ x)) ◇ w) ◇ (w ◇ ((x ◇ ((y ◇ x) ◇ x)) ◇ w))) ◇ ((x ◇ ((y ◇ x) ◇ x)) ◇ (((x ◇ ((y ◇ x)
            ◇ x)) ◇ w) ◇ (w ◇ ((x ◇ ((y ◇ x) ◇ x)) ◇ w)))))))))) =
        (((x ◇ ((y ◇ x) ◇ x)) ◇ z) ◇ (((x ◇ ((y ◇ x) ◇ x)) ◇ w) ◇ (((x ◇ ((y ◇ x) ◇ x)) ◇ ((x ◇ ((y ◇ x) ◇ x)) ◇ w)) ◇
          (w ◇ ((x ◇ ((y ◇ x) ◇ x)) ◇ w))))) := by
        duper [op_law]
      (((x ◇ ((y ◇ x) ◇ x)) ◇ z) ◇ (((x ◇ ((y ◇ x) ◇ x)) ◇ w) ◇ (((x ◇ ((y ◇ x) ◇ x)) ◇ ((x ◇ ((y ◇ x) ◇ x)) ◇ w)) ◇
          (w ◇ ((x ◇ ((y ◇ x) ◇ x)) ◇ w))))) =
        (((x ◇ ((y ◇ x) ◇ x)) ◇ z) ◇ (((x ◇ ((y ◇ x) ◇ x)) ◇ w) ◇ ((x ◇ ((y ◇ x) ◇ x)) ◇ ((x ◇ ((y ◇ x) ◇ x)) ◇ w)))) := by
        duper [lemma_7]
      (((x ◇ ((y ◇ x) ◇ x)) ◇ z) ◇ (((x ◇ ((y ◇ x) ◇ x)) ◇ w) ◇ ((x ◇ ((y ◇ x) ◇ x)) ◇ ((x ◇ ((y ◇ x) ◇ x)) ◇ w)))) =
        ((x ◇ ((y ◇ x) ◇ x)) ◇ z) := by
        duper [lemma_7]

  have lemma_9 (x y z : G) :
  (x ◇ ((y ◇ ((z ◇ y) ◇ y)) ◇ x)) = x := by
    calc
      (x ◇ ((y ◇ ((z ◇ y) ◇ y)) ◇ x)) =
        (x ◇ (((y ◇ ((z ◇ y) ◇ y)) ◇ x) ◇ ((y ◇ ((z ◇ y) ◇ y)) ◇ x))) := by
        duper [lemma_8]
      (x ◇ (((y ◇ ((z ◇ y) ◇ y)) ◇ x) ◇ ((y ◇ ((z ◇ y) ◇ y)) ◇ x))) =
        (x ◇ (((y ◇ ((z ◇ y) ◇ y)) ◇ x) ◇ (((y ◇ ((z ◇ y) ◇ y)) ◇ x) ◇ ((y ◇ ((z ◇ y) ◇ y)) ◇ x)))) := by
        duper [lemma_8]
      (x ◇ (((y ◇ ((z ◇ y) ◇ y)) ◇ x) ◇ (((y ◇ ((z ◇ y) ◇ y)) ◇ x) ◇ ((y ◇ ((z ◇ y) ◇ y)) ◇ x)))) =
        x := by
        duper [op_law]

  show _ by
    intros x y z
    calc
      x = (x ◇ ((x ◇ ((x ◇ x) ◇ x)) ◇ x)) := by
        duper [lemma_9]
      (x ◇ ((x ◇ ((x ◇ x) ◇ x)) ◇ x)) =
        ((x ◇ ((x ◇ ((x ◇ x) ◇ x)) ◇ x)) ◇ ((y ◇ (z ◇ (x ◇ z))) ◇ ((x ◇ ((x ◇ x) ◇ x)) ◇ (y ◇ (z ◇ (x ◇ z)))))) := by
        duper [lemma_5]
      ((x ◇ ((x ◇ ((x ◇ x) ◇ x)) ◇ x)) ◇ ((y ◇ (z ◇ (x ◇ z))) ◇ ((x ◇ ((x ◇ x) ◇ x)) ◇ (y ◇ (z ◇ (x ◇ z)))))) =
        (x ◇ ((y ◇ (z ◇ (x ◇ z))) ◇ ((x ◇ ((x ◇ x) ◇ x)) ◇ (y ◇ (z ◇ (x ◇ z)))))) := by
        duper [lemma_9]
      (x ◇ ((y ◇ (z ◇ (x ◇ z))) ◇ ((x ◇ ((x ◇ x) ◇ x)) ◇ (y ◇ (z ◇ (x ◇ z)))))) =
        (x ◇ (y ◇ (z ◇ (x ◇ z)))) := by
        duper [lemma_9]
```

Vampire

Vampire is a saturation-based theorem prover for first- and higher-order logic with equality.

It is developed by Andrei Voronkov and colleagues.

It is based on the superposition calculus and implements highly optimized search strategies and data structures.

It finished first in the unit equality division of CASC 2025.

Vampire Proofs

Vampire proofs are refutational.
They derive $\bot$ from the axioms and the negated conjecture.

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They derive $\bot$ from the axioms and the negated conjecture.

Example:

1. $a = b$	axiom
2. $f(x) = x$	axiom
3. $h(f(b)) \neq h(a)$	negated conjecture
4. $h(b) \neq h(a)$	from 2 and 3
5. $h(a) \neq h(a)$	from 1 and 4
6. $\bot$	from 5

Redirecting Vampire Proofs

We introduce quantifiers and apply the contrapositive to get positive equations.

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Before:

$$\frac{\mathsf{h}(\mathsf{a}, y) = \mathsf{b} \qquad \mathsf{h}(x, \mathsf{a}) \neq x}{\mathsf{b} \neq \mathsf{a}}$$

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Before:

$$\frac{\mathsf{h}(\mathsf{a}, y) = \mathsf{b} \qquad \mathsf{h}(x, \mathsf{a}) \neq x}{\mathsf{b} \neq \mathsf{a}}$$

With quantifiers:

$$\frac{\forall y.\ \mathsf{h}(\mathsf{a}, y) = \mathsf{b} \qquad \forall x.\ \mathsf{h}(x, \mathsf{a}) \neq x}{\mathsf{b} \neq \mathsf{a}}$$

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With quantifiers:

$$\frac{\forall y.\ \mathsf{h}(\mathsf{a}, y) = \mathsf{b} \qquad \forall x.\ \mathsf{h}(x, \mathsf{a}) \neq x}{\mathsf{b} \neq \mathsf{a}}$$

After contrapositive:

$$\frac{\forall y.\ \mathsf{h}(\mathsf{a}, y) = \mathsf{b} \qquad \mathsf{b} = \mathsf{a}}{\exists x.\ \mathsf{h}(x, \mathsf{a}) = x}$$

Redirecting Vampire Proofs

Before:

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2.	$f(x) = x$	axiom
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6.	$\bot$	from 5

After:

1.	$a = b$	axiom
2.	$\forall x.\ f(x) = x$	axiom
3.	$h(b) = h(a)$	from 1 and $h(a) = h(a)$
4.	$h(f(b)) = h(a)$	from 2 and 3

Twee

Twee is an automatic prover specialized for equational reasoning.

It is developed by Nicholas Smallbone.

It is based on the unfailing completion procedure,
an extension of Knuth–Bendix completion.

It finished second in the unit equality division of CASC 2025.

Twee Proofs

Twee proofs consist of a sequence of lemmas,
where the lemmas and the conjecture are proved by chains of equalities.

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where the lemmas and the conjecture are proved by chains of equalities.

Example:

Axiom 1: $a = b$

Axiom 2: $f(x) = x$

Lemma 3: $f(b) = a$
Proof:
$f(b)$
$=$ { by axiom 1 right-to-left }
$f(a)$
$=$ { by axiom 2 }
a

Goal 1: $h(f(b)) = h(a)$
Proof:
$h(f(b))$
$=$ { by lemma 3 }
$h(a)$

Proof Generation for Intermediate Lemmas

Theorem

$A \Longrightarrow C$

Vampire Proof

A	axiom
L_1	from A and A
L_2	from A and L_1
C	from L_1 and L_2

Proof Generation for Intermediate Lemmas

Theorem

$A \Longrightarrow C$

Vampire Proof

A	axiom
L_1	from A and A
L_2	from A and L_1
C	from L_1 and L_2

For each intermediate lemma, we generate three problem variants:

1. big-step problem,
2. small-step problem,
3. abstracted problem.

Each problem is given to Vampire and Twee.

Big-Step Problems

Big-step problems contain only the axioms and the lemma as the conjecture.

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Problem for L_1

A	axiom
L_1	conjecture

Problem for L_2

A	axiom
L_2	conjecture

Small-Step Problems

Small-step problems contain the axioms, the lemma as the conjecture, and all previously derived lemmas as additional axioms.

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A	axiom
L_1	conjecture

Problem for L_2

A	axiom
L_1	axiom
L_2	conjecture

Abstracted Problems

Abstracted problems contain the axioms and an abstracted version of the lemma where some subterms are replaced with fresh variables.

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Problem for L_1

A	axiom
L_1'	conjecture

Problem for L_2

A	axiom
L_2'	conjecture

Proof Construction for the Main Theorem

The minimized proof consists of three segments:

1. from the axioms to a *departure lemma*,
2. from the departure lemma to an *arrival lemma*,
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We identify up to six intermediate steps near the end of the Vampire proof as candidate arrival lemmas.

For each candidate arrival lemma, its transitive dependencies are considered as candidate departure lemmas.

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We identify up to six intermediate steps near the end of the Vampire proof as candidate arrival lemmas.

For each candidate arrival lemma, its transitive dependencies are considered as candidate departure lemmas.

Why three segments? It constitutes a trade-off between performance and flexibility.

Proof Construction for the Main Theorem

For each segment:

1. We create a problem.
2. We run Vampire and Twee.
3. We keep the shortest proof.

Proof Construction for the Main Theorem

For each segment:

1. We create a problem.
2. We run Vampire and Twee.
3. We keep the shortest proof.

To obtain the final minimized proof, we concatenate the three segments, pruning unreferenced lemmas.

Example of Our Approach

Consider this 7-step Vampire proof of $A \Longrightarrow C$, which we want to minimize:

A	axiom
L_1	from A and A
L_2	from A and L_1
L_3	from L_1 and L_2
L_4	from L_2 and L_3
L_5	from L_3 and L_4
L_6	from A and L_5
C	from L_5 and L_6

Example of Our Approach

Consider this 7-step Vampire proof of $A \Longrightarrow C$, which we want to minimize:

A axiom
L_1 from A and A
L_2 from A and L_1
L_3 from L_1 and L_2
L_4 from L_2 and L_3
L_5 from L_3 and L_4
L_6 from A and L_5
C from L_5 and L_6

For each lemma, we create the problem variants and try to prove them, keeping the shortest proof.

Example of Our Approach

Steps $L_2, \ldots, L_6$ and C are considered as candidate arrival lemmas.
Let's focus on L_6. Here is the shortest proof of L_6:

A axiom
L_1 from A and A
L_2 from A and L_1
L_3 from L_1 and L_2
L_4 from L_2 and L_3
L_5 from L_3 and L_4
L_6 from A and L_5

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L_4 from L_2 and L_3
L_5 from L_3 and L_4
L_6 from A and L_5

Lemmas L_1 to L_5 are considered as candidate departure lemmas.
Let's focus on L_3.

Example of Our Approach

Segment 1:

We create a problem with A, L_1, and L_2 as axioms and L_3 as the conjecture.
The shortest proof is found by Vampire:

A axiom
L_1 from A and A
L_2' from A and L_1
L_3 from L_1 and L_2'

Example of Our Approach

Segment 2:

We create a problem with A, L_1, L_2', and L_3 as axioms and L_6 as the conjecture.
The shortest proof is found by Twee:

L_6 by a 2-step equality chain using L_1 and L_3

Example of Our Approach

Segment 3:

We create a problem with A, L_1, L_2', L_3, and L_6 as axioms and C as the conjecture.
The shortest proof is found by Twee:

C by a 2-step equality chain using L_2' and L_3

Since this proof does not use the arrival lemma L_6, we skip segment 2.

Example of Our Approach

Final Proof:

A axiom
L_1 from A and A
L_2' from A and L_1
L_3 from L_1 and L_2'
C by a 2-step equality chain using L_2' and L_3

Constructing Segment 1

In general, to prove the departure lemma:

1. We create a problem with the departure lemma's dependencies (according to the arrival lemma's proof) as axioms and the departure lemma as the conjecture.
2. We run Vampire and Twee.
3. We keep the shorter proof.

Constructing Segment 1

In general, to prove the departure lemma:

1. We create a problem with the departure lemma's dependencies (according to the arrival lemma's proof) as axioms and the departure lemma as the conjecture.
2. We run Vampire and Twee.
3. We keep the shorter proof.

This proof, together with the shortest proofs of its dependencies, forms segment 1, unless a shorter proof was found earlier for one of the problem variants.

Constructing Segment 2

In general, to prove the arrival lemma:

1. We create a problem with the departure lemma and its dependencies as axioms and the arrival lemma as the conjecture.
2. We run Vampire and Twee.
3. We keep the shorter proof.

This proof, together with the shortest proofs of its dependencies, forms segment 2, unless a shorter proof was found earlier for one of the problem variants.

Constructing Segment 3

In general, to prove the main conjecture:

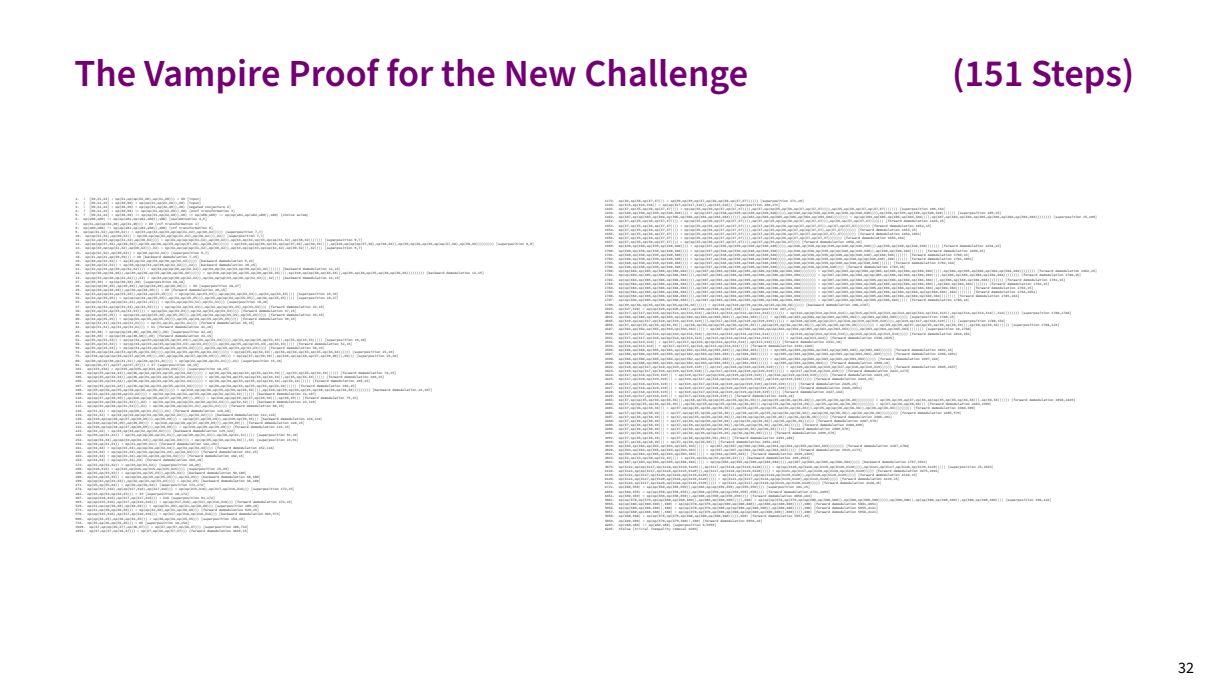
1. We create a problem with the departure lemma, the arrival lemma, and their dependencies as axioms and the main conjecture as the conjecture.
2. We run Vampire and Twee.
3. We keep the shorter proof.

This proof, together with the shortest proofs of its dependencies, forms segment 3, unless a shorter proof was found earlier for one of the problem variants.

Another Challenge

$$\begin{aligned} & \left(\forall x, y, z.\ x = y \diamond ((z \diamond x) \diamond (y \diamond x))\right) \\ \Longrightarrow\ & \left(\forall x, y, z.\ x \diamond x = (y \diamond (z \diamond x)) \diamond x\right) \end{aligned}$$

The Vampire Proof for the New Challenge (151 Steps)



Krympa's Proof for the New Challenge (10 Steps)

```
class Magma (α : Type _) where
  op : α → α → α

infix:65 " ∘ " => Magma.op

abbrev Equation_a1 (G : Type _) [Magma G] :=
  ∀ x y z : G, x = (y ∘ ((z ∘ x) ∘ (y ∘ x)))

abbrev Equation_conjecture0 (G : Type _) [Magma G] :=
  ∀ x y z : G, (x ∘ x) = ((y ∘ (z ∘ x)) ∘ x)

theorem Equation_a1_implies_Equation_conjecture0 (G : Type _) [Magma G]
(op_law : Equation_a1 G) : Equation_conjecture0 G :=
  have lemma_1 (x y z w : G) :
  ((x ∘ y) ∘ (z ∘ y)) = (w ∘ (y ∘ (w ∘ ((x ∘ y) ∘ (z ∘ y))))) := by
    duper [op_law]

  have lemma_2 (x y z : G) :
  ((x ∘ y) ∘ (z ∘ y)) = (z ∘ (y ∘ y)) := by
    duper [lemma_1, op_law]

  have lemma_3 (x y : G) :
  (x ∘ (x ∘ (y ∘ y))) = y := by
    duper [op_law, lemma_2]

  have lemma_4 (x y z : G) :
  (x ∘ x) = ((y ∘ (z ∘ x)) ∘ x) := by
    calc
      (x ∘ x) = ((y ∘ (z ∘ x)) ∘ ((y ∘ (z ∘ x)) ∘ ((x ∘ x) ∘ (x ∘ x)))) := by
        duper [lemma_3]
      ((y ∘ (z ∘ x)) ∘ ((y ∘ (z ∘ x)) ∘ ((x ∘ x) ∘ (x ∘ x)))) =
        ((y ∘ (z ∘ x)) ∘ ((y ∘ (z ∘ x)) ∘ (z ∘ (z ∘ (((x ∘ x) ∘ (x ∘ x)) ∘ ((x ∘ x) ∘ (x ∘ x)))))) := by
        duper [lemma_3]
      ((y ∘ (z ∘ x)) ∘ ((y ∘ (z ∘ x)) ∘ (z ∘ (z ∘ (((x ∘ x) ∘ (x ∘ x)) ∘ ((x ∘ x) ∘ (x ∘ x))))))) =
        ((y ∘ (z ∘ x)) ∘ ((y ∘ (z ∘ x)) ∘ (z ∘ (z ∘ ((x ∘ x) ∘ ((x ∘ x) ∘ (x ∘ x))))))) := by
        duper [lemma_2]
      ((y ∘ (z ∘ x)) ∘ ((y ∘ (z ∘ x)) ∘ (z ∘ (z ∘ ((x ∘ x) ∘ ((x ∘ x) ∘ (x ∘ x)))))) =
        ((y ∘ (z ∘ x)) ∘ ((y ∘ (z ∘ x)) ∘ (z ∘ (z ∘ x)))) := by
        duper [lemma_3]
      ((y ∘ (z ∘ x)) ∘ ((y ∘ (z ∘ x)) ∘ (z ∘ (z ∘ x)))) =
        ((y ∘ (z ∘ x)) ∘ (z ∘ ((z ∘ x) ∘ (z ∘ x)))) := by
        duper [lemma_2]
      ((y ∘ (z ∘ x)) ∘ (z ∘ ((z ∘ x) ∘ (z ∘ x)))) = ((y ∘ (z ∘ x)) ∘ (z ∘ (z ∘ (x ∘ x)))) := by
        duper [lemma_2]
      ((y ∘ (z ∘ x)) ∘ (z ∘ (z ∘ (x ∘ x)))) = ((y ∘ (z ∘ x)) ∘ x) := by
        duper [lemma_3]

  show _ by
    exact lemma_4
```

Evaluation on the Equational Theories Project

(Vampire Proofs of at Least 15 Steps)

File	Num. problems	Avg. before	Avg. after			
			BA	SA	BS	BSA
`Proofs1.lean`	135	17.3	16.0	**13.1**	13.3	13.3
`Proofs2.lean`	117	16.9	14.4	**11.5**	**11.5**	**11.5**
`Proofs3.lean`	108	19.3	15.6	10.9	**10.7**	10.9
`Proofs4.lean`	125	19.1	14.3	**10.9**	11.4	11.4
`Proofs5.lean`	116	20.1	17.6	12.8	**11.9**	**11.9**
`Proofs6.lean`	115	25.6	18.9	**12.5**	12.6	12.6
`Proofs7.lean`	117	37.2	19.7	**11.8**	11.9	11.9
`Proofs8.lean`	114	24.4	15.6	**12.3**	13.1	13.1
`Proofs9.lean`	112	39.8	29.0	**13.1**	14.1	14.1
`Proofs10.lean`	101	21.5	16.0	**8.0**	11.0	11.0
`Proofs11.lean`	110	25.4	22.4	**13.0**	14.0	14.0
`Proofs12.lean`	123	24.6	16.5	**8.0**	8.5	8.5
`Proofs13.lean`	38	35.3	27.7	**9.1**	10.1	10.1

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Related Work

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Mikoláš Janota empirically found that Vampire's superposition calculus together with finite model finding can solve almost all Equational Theories Project problems.

Vampire's success might be partly due to the work on term ordering diagrams by Márton Hajdu et al.

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Alex Gu et al. developed ProofOptimizer, which uses large language models to simplify Lean proofs.

Conclusion

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Proof automation and readability can go hand in hand.

Future Work

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We could try to translate Vampire proofs to Twee's equality chain format.

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We could try more provers.

Tao's Equational Proof Challenge Accepted

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