Enumerating Choice Terms in Model-Based Quantifier Instantiation

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Satisfiability Modulo Theories (SMT) Solvers

SMT solvers combine a SAT solver with decision procedures for theories.

They work by refutation: Assume the negation of the conjecture and derive $\bot$.

They are usually based on first-order logic, but some support higher-order logic.

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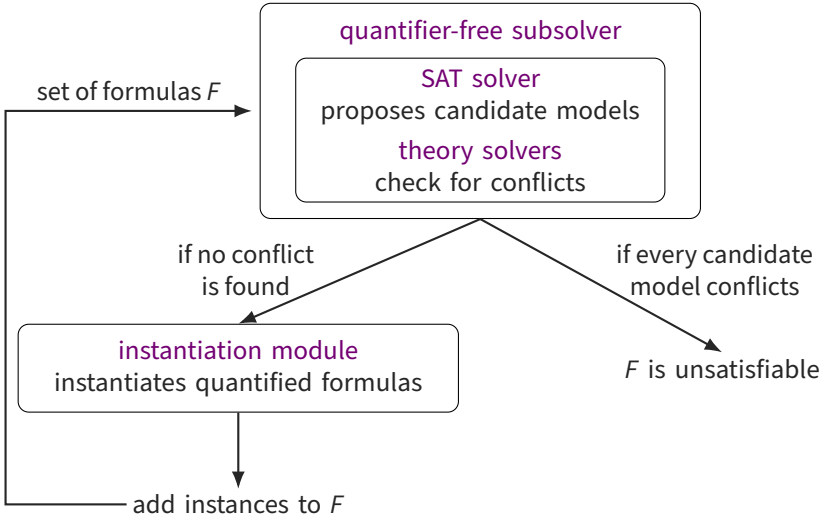

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Existential quantifiers are handled by Skolemization.

MBQI-Enum

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Our work in cvc5, MBQI-Enum, extends MBQI with a SyGuS-based enumerator.

Key idea:

Instead of relying only on terms already present in the model,

generate new candidate terms from a grammar built from symbols in the problem,

including λ-abstractions for variables of function type.

MBQI-Enum Example

Consider the input problem

$$F = \{ \forall y.\ \exists z.\ y\, z \neq \mathsf{f}\, z \},$$

where $\mathsf{f} : u \to u$ is a function symbol and $y : u \to u$ is a variable.

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After Skolemization, the quantifier-free subsolver finds a contradiction.

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ε terms can be the right instantiations even when they do not occur in the input problem.

MBQI-Enum with Choice

Our strategy extends MBQI-Enum by:

- adding ε-terms to the grammar;
- introducing fresh Skolem symbols to represent these terms;
- filtering redundant or invalid candidates; and
- returning a substitution and also auxiliary lemmas characterizing the new Skolems.

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5. Return σ together with lemmas that characterize the generated choice terms.

Example of MBQI-Enum with Choice

Let $\mathsf{f} : u \to u$ be an arbitrary function.

Proof goal: if f is injective, then it has a left inverse, that is,

$$(\forall x, y.\ \mathsf{f}\, x = \mathsf{f}\, y \Longrightarrow x = y) \Longrightarrow \exists g.\ \forall z.\ g\,(\mathsf{f}\, z) = z.$$

Example of MBQI-Enum with Choice

We consider the set $F = \{\forall x, y.\, \mathsf{f}\, x = \mathsf{f}\, y \Longrightarrow x = y,\ \forall g.\, \exists z.\, g\,(\mathsf{f}\, z) \neq z\}$.

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The strategy generates

$$\lambda y.\ \varepsilon x.\ y = \mathsf{f}\, x.$$

The ε subterm is abstracted by a fresh Skolem $\mathsf{h} : u \to u$, yielding

$$\lambda y.\ \mathsf{h}\, y.$$

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Augmented Grammar

For a variable of function type y_i, the grammar contains rules such as

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The final extended grammar is

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The goal is to abstract this ε term into a form that the SMT solver can reason about.

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We construct the lemma

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The rewritten term is used in the substitution.

Term Filtering

Redundant candidates are discarded, so equivalent terms are considered only once.

For $\varepsilon x.\ \varphi$, the bound variable x must occur in φ; otherwise, the term is excluded.

cvc5 already performs extensive theory-specific filtering.

Evaluation

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Vampire					
Zipperposition					
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- We evaluated on 3757 higher-order problems from TPTP with a 60s timeout.

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- Our approach solves the largest number of problems (2382), 24 more than the next-best prover.
- Compared with the baseline, our approach solves 23 additional unsatisfiable benchmarks.
- TPTP contains very few problems needing choice; most are choice-free, so the effect of choice support is hard to assess.

Conclusion

We extended MBQI-Enum with support for Hilbert's choice.

This makes it possible to solve problems containing choice,
but also problems where the right instantiation is an ε term.

We implemented the approach in the SMT solver cvc5.

Future Work

A main limitation is that useful ε-terms often appear too late in enumeration.

Since enumeration proceeds by increasing term size, these terms may be too complex.

Better enumeration heuristics could help generate the right terms earlier.

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